7. $\sum_{n=0}^{\infty} \frac{(n+1)(2x+1)^n}{(2n+1)2^n}$

ratio: $\lim_{n\to\infty} \left| \frac{(n+2)(2x+1)^{\cancel{n+1}}}{(2(n+1)+1)2^{\cancel{n+1}}} \cdot \frac{(2n+1)\cancel{2^n}}{(n+1)\cancel{(2x+1)^n}} \right| = \frac{|2x+1|}{2} < 1$

$\to \frac{1}{2}$ $\to 2$

$|2x+1| < 2$ radius $= 1$

$-2 < 2x+1 < 2$

$-3 < 2x < 1$

$\boxed{-\frac{3}{2} < x < \frac{1}{2}}$ ← interval

$-3/2 \quad -1/2 \quad 1/2$

endpts. $x = -\frac{3}{2}$ $\sum \frac{(n+1)(-2)^n}{(2n+1)2^n}$

diverges $\sum \frac{n+1}{2n+1}(-1)^n$

$a_n \to \frac{1}{2}$

$x = \frac{1}{2}$ $\sum \frac{n+1}{2n+1} \frac{2^n}{2^n}$

diverges

$a_n \to \frac{1}{2}$

conv. absolutely in $\left(-\frac{3}{2}, \frac{1}{2}\right)$

conv conditionally: no where

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13. $\sum_{n=1}^{\infty} \frac{n!}{2^n} \cdot x^{2n}$

ratio: $\lim_{n\to\infty} \left| \frac{(n+1)\cancel{!}\, x^{2n+2}}{2^{\cancel{n+1}}} \cdot \frac{\cancel{2^n}}{\cancel{n!}\,\cancel{x^{2n}}} \right| = \lim_{n\to\infty} \left| \frac{n+1}{2} x^2 \right|$

$= \infty$

diverges for all x except $x = 0$

$\frac{(n+1)!}{n!} = \frac{(n+1)\cdot\cancel{n(n-1)\cdots 1}}{\cancel{n(n-1)\cdots 1}}$

radius $= 0$

IOC $x = 0$

conv absolutely at $x = 0$

Cond. conv. no where

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33. $xe^{-x^2} = x - x^3 + \frac{x^5}{2} - \frac{x^7}{3!} \ldots (-1)^n \frac{x^{2n+1}}{n!} \ldots$

$e^{x} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} \ldots \frac{x^n}{n!}$

$\frac{(-x^2)^n}{n!}$

sub

$e^{-x^2} = 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{3!} \ldots (-1)^n \frac{x^{2n}}{n!}$

e^{2x+1}

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56. $P_4(x) = 7 - 3(x-4) + 5(x-4)^2 - 2(x-4)^3 + 6(x-4)^4$

$a=4$

$P_4(x) = f(4) + f'(4)(x-4) + \frac{f''(4)}{2}(x-4)^2 + \frac{f'''(4)}{3!}(x-4)^3 + \frac{f^{4}(4)}{4!}(x-4)^4$

a) $f(4) = 7$ $\quad \frac{f'''(4)}{\cancel{3!}} = -2 \cdot 3! = -12$

b) 2^{nd} order $\quad p'(x) = -3 + 2\cdot 5(x-4) - 3\cdot 2(x-4)^2$

$f(4.3) \approx -3 + 10(.3) - 6(.3)^2 = \_\_\_$

c) $\int_4^x f(t)\,dt = 7t - \frac{3(t-4)^2}{2} + \frac{5(t-4)^3}{3} - \frac{2(t-4)^4}{4} \Big|_4^x$

$= 7x - \frac{3}{2}(x-4)^2 + \frac{5}{3}(x-4)^3 - \frac{2}{4}(x-4)^4 - 7\cdot 4$

d) $f(3)$ exact? no taylor's series approx $f(x)$ for x near 4

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